Excitations of vortices in Abelian Higgs model

1997 J. Phys. A: Math. Gen. 30 1191

(http://iopscience.iop.org/0305-4470/30/4/019)

J. Phys. A: Math. Gen. **30** (1997) 1191–1198. Printed in the UK PII: S0305-4470(97)64889-1

Excitations of vortices in Abelian Higgs model

T Dobrowolski
Institute of Physics and Informatics WSP, Podchorążych 2, 30-084 Kraków, Poland

Received 15 May 1995, in final form 16 October 1996

Abstract. The method of constructing excited vortex solutions with higher than unit winding numbers is used to investigate bubble-like excitations of a closed loop and the $\frac{\pi}{2}$ scattering.

1. Introduction

Recently there has been a wide interest in the study of nonlinear field equations which contain vortex solutions. The reason for this interest is their presence in many contexts in cosmology [1], particle physics [2] and condensated matter physics [3]. As yet they were worked out in two general ways of describing dynamics of vortices. The first is an effective action method which, although correct in principle, can yield rather unpleasant effective theories. In a generic case one should expect nonlocal effective theories containing higher derivatives and requiring a restriction on initial data. The restriction on initial data is necessary for equivalence with the original model [4].

Secondly is a family of methods which does not use an effective action. They are free from the shortcomings presented in the first method. The second method, as well as the first method, leads to reduction of the dynamics of the vortex to the dynamics of the string. The idea of such a reduction of degrees of freedom has its origin in the Nielsen and Olesen article [5]. Their observation is based on the property of the vortex solutions that when we approach some values of parameters of these solutions the width of the vortex (understood as the width of the region in space where density of the energy significantly differs from its vacuum value) decreases—finally reaching zero width. In this limit vortex-tube coincides with the line of zeros of the Higgs field. This is the reason for regarding the vortex as a string in the above mentioned limit. This idea has been generalized by Förster to describe nontrivial time evolution of vortices [6].

The string description, although not exact as it is in the zero width limit, is possible for vortices with non-zero width. If we define string, for example, as a line of zeros of the Higgs field, we will obtain an approximate string description of vortex.

During the time evolution the line of zeros of the Higgs field sweeps out a two-dimensional manifold which is called the world-sheet of the vortex. Such a definition of the world-sheet has the same shortcoming as the definition of the phase speed in the description of a moving wave. It has been shown that when two vortices interact with each other the speed of the lines of zeros of the Higgs field can exceed the velocities of light [7]. As we know the physical aspects of a moving wave are better described by the group speed which never exceeds the speed of light. If we define the world-sheet of a vortex as a surface swept out by a set of points where the density of the energy of the vortex has its maximal value E_{max}, we will obtain the characteristic similar to the group speed in the

0305-4470/97/041191+08$19.50

problem of a moving wave. Such a physical description works very well for one-quantum flux vortex solutions but for solutions with a winding number bigger than one it can be saved only in case when we define a string as a collection of 'centres-of-mass' of the vortex [8] (vortices with $n > 1$ are stable for coupling constant smaller than the critical value). This difficulty is a consequence of the fact that for vortex with $n \geqslant 2$ the maximum of the density of the energy E does not lie on a line as it is for vortex with $n = 1$ but it lies on the surface enclosing the line of zeros of the Higgs field [9].

Although the string description of the vortex solutions with $n \geqslant 2$ exists it seems that in this case the membrane description could be more suitable and more precise [10]. Higher precision in the case of vortices with a winding number bigger than one has purely geometrical nature—a membrane has a reacher geometrical structure than a string. The membrane method does not give a comprehensive description of vortices with $n \geqslant 2$, but it only gives a description of some special, although very important from physical point of view, property of the field configuration, namely the density of the energy. The membrane description could be compared with a string description of vortices based on a surface of the zeros of the Higgs field. As it is known a surface of the zeros of the Higgs field, independently of the finite thickness of the vortex, is a well-defined geometrical object in spacetime. Similarly, the hypersurface of the maximum of the energy density is also well defined. The difference between the surface of the zeros of the Higgs field and the hypersurface of the maximum of the density of the energy lies in fact that only the latter has a well-defined physical meaning while the surface of zeros is a more mathematical concept (e.g. it can move with the velocity greater than a velocity of light).

The comprehensive membrane description of vortices in Abelian Higgs model does not exist, in this sense that does not exist the membrane limit of vortices, although the string limit of vortices exists. Nonexistence of the membrane limit of the vortex is a consequence of the fact that the density of the energy does not vanish at the line of zeros of the Higgs field.

The paper is organized as follows. In section 2 we fix our notation and remind the membrane method of construction of the excited vortex solutions. Excitations of the vortex loop are investigated in section 3. Section 4 is devoted to looking for the tracks of the $\frac{\pi}{2}$ scattering in the membrane method and section 5 contains remarks.

2. Reminding of the membrane method of construction excited vortex solutions

At each instant of time we identify the membrane with a surface $\partial_\xi \omega|_{\xi=0} = 0$ where $\omega = g^{ab} X^\mu{}_{,a} X^\nu{}_{,b} T_{\mu\nu}$ (see notation below). Projection $T_{\mu\nu}$ on hypersurface X^μ ensures equality $\partial_\xi \omega|_{\xi=0} = \partial_\xi E|_{\xi=0}$ for static cylindrically symmetric solutions, where E denotes the density of the energy. Moreover, for slightly perturbed static solutions the difference between hypersurfaces defined by both conditions is also small. This is reason for calling $\partial_\xi \omega|_{\xi=0} = 0$ 'maximum of the density of the energy condition'. Time evolution of this surface yields a three-dimensional manifold in four-dimensional spacetime. A radius-vector of an arbitrary point on the world-hypersurface $X^\mu(\sigma^a)$, where $\mu = 0, 1, 2, 3$ are lorentzian indices, can be parameterized by the time-like parameter τ, length-like parameter σ and parameter θ which can be identified with an angle of cylindrical coordinates. The coordinates $(\tau, \sigma, \theta) = (\sigma^a)$ are variables on this three-dimensional manifold. The coordinate on the straight line perpendicular to the world-hypersurface Σ at the point $X^\mu(\tau, \sigma, \theta)$ we mark ξ. Having all the variables $(\zeta^\alpha) = (\sigma^\alpha, \xi)$ in the neighbourhood of the world-hypersurface Σ

we can write

$$x^{\mu} = X^{\mu}(\tau, \sigma, \theta) + n^{\mu}\xi \tag{1}$$

where n^{μ} is four-vector normal to the hypersurface Σ in the point $X^{\mu}(\sigma^{a})$. Tangent four-vectors $X^{\mu}{}_{,a} = \partial_a X^{\mu}$, and normal four-vector $n_i{}^{\mu}$ to the hypersurface Σ obey the conditions:

$$n^{\mu}n_{\mu} = -1 \qquad n_{\mu}X^{\mu}{}_{,a} = 0 \qquad g_{ab} = X^{\mu}{}_{,a}X_{\mu,b} \tag{2}$$

where $\eta^{\mu\nu}$ is Minkowski metric with signature $(+,-,-,-)$. The last equation is a definition of the intrinsic metric on the world-hypersurface Σ. Coefficients K_{ab} of extrinsic curvature are defined by the equations $K_{ab} = -X^{\mu}{}_{,ab}n_{\mu}$.

The construction of a membrane theory describing the sector of the Abelian Higgs model containing small perturbations of any fixed vortex solution with $n \geqslant 2$ starts from the Euler–Lagrange equations for lagrangian density:

$$L = (D_{\mu}\phi)^{*}(D^{\mu}\phi) - \frac{1}{4}\lambda\left(\phi^{*}\phi - \frac{2m^{2}}{\lambda}\right)^{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{3}$$

where $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is the field strength, $D_{\mu}\phi = \partial_{\mu}\phi - \mathrm{i}eA_{\mu}\phi$ is covariant derivative, ϕ is a complex scalar field and A_{μ} is a gauge potential.

An excited vortex is naturally obtained as a solution to equations of motion in coordinates on the curved hypersurface. We assume that

$$\begin{aligned}
&\phi = \phi(\tau, \sigma, \theta, r_0(\tau, \sigma, \theta) + \xi) \xrightarrow[\xi\,\text{sufficiently large}]{} \mathrm{e}^{-in\theta}F(\xi)\\
&A_a = A_a(\tau, \sigma, \theta, r_0(\tau, \sigma, \theta) + \xi) \qquad A_{\xi} = A_{\xi}(\tau, \sigma, \theta, r_0(\tau, \sigma, \theta) + \xi)
\end{aligned} \tag{4}$$

are unknown vortex solutions (with $n \geqslant 2$) of field equations. On the other hand, ansatz (4) could be interpreted as a perturbation of the static solution obtained by local scalings and shifts of the fields. The radius $r_0(\tau, \sigma, \theta)$ of a membrane of maximum of the density of the energy for excited vortex can be split into the radius of a membrane r_0 (describing the maximum of the density of the energy of the unperturbed vortex) and the small perturbation $\varepsilon(\tau, \sigma, \theta)$, i.e. $r_0(\tau, \sigma, \theta) = r_0 + \varepsilon(\tau, \sigma, \theta)$. In the next step, assuming that small perturbation of functions causes the same order perturbation of shape of the cylinder (i.e. coefficients of extrinsic curvature K_{ab}), we expand equations of motion with respect to powers of ε.

The first-order equations of expansion of the equations of motion in perturbation ε have the form

$$\phi^{1}k_{a}{}^{a} = 0 \tag{5}$$

$$g^{ab}\nabla_a(k_b{}^{c}A^{0}{}_{c}) - 2k^{ab}K^{(0)}{}_{ab}A^{0}{}_{\xi} = 0 \tag{6}$$

$$\begin{aligned}
&g^{ab}\nabla_a(k_{bh}A^{0}{}_{\xi}) - k_h{}^{d}A^{1}{}_{d} + k_h{}^{c}(\partial_c A^{0}{}_{\xi} - A^{1}{}_{c})\\
&\qquad + [2k_{hc}K_{(0)}{}^{cd} + k_h{}^{d}K_{(0)}{}_{c}{}^{c} + 2k^{ad}K^{(0)}{}_{ah}]A^{0}{}_{d} = 0
\end{aligned} \tag{7}$$

$$\begin{aligned}
&k^{ab}[g^{cd}(\partial_a A^{0}{}_{c} - \partial_c A^{0}{}_{a})(\partial_b A^{0}{}_{d} - \partial_d A^{0}{}_{b}) + (\partial_a A^{0}{}_{\xi} - A^{1}{}_{a})(\partial_b A^{0}{}_{\xi} - A^{1}{}_{b})\\
&\qquad + (\partial_a\phi^{0} - \mathrm{i}eA^{0}{}_{a}\phi^{0})^{*}(\partial_b\phi^{0} - \mathrm{i}eA^{0}{}_{b}\phi^{0})] = 0.
\end{aligned} \tag{8}$$

Equations (5)–(8) form the effective membrane theory which have internal degrees of freedom given by the $A^{0}{}_{\xi}$, $A^{0}{}_{a}$, $A^{1}{}_{a}$, ϕ^{0}, ϕ^{1}, which are defined by the expansions

$$\begin{aligned}
&\phi = \phi^{0} + \xi\phi^{1} + \xi^{2}\phi^{2} + \cdots\\
&A_a = A^{0}{}_{a} + \xi A^{1}{}_{a} + \xi^{2}A^{2}{}_{a} + \cdots \qquad A_{\xi} = A^{0}{}_{\xi} + \xi A^{1}{}_{\xi} + \xi^{2}A^{2}{}_{\xi} + \cdots
\end{aligned} \tag{9}$$

(where coefficients of expansions are functions of (σ^{a})). In first-order approximation coefficients $A^{0}{}_{\xi}$, $A^{0}{}_{a}$, $A^{1}{}_{a}$, ϕ^{0}, ϕ^{1}, as the only fields defined on the hypersurface X^{μ},

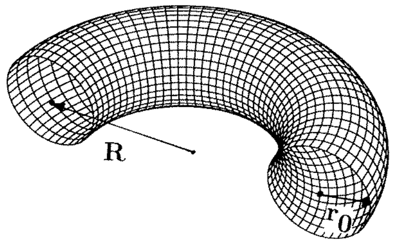

Figure 1. Toroidal surface is described by radius of the loop R and radius r_0.

describe at least part of the interactions between the walls of the membrane present in the original theory. Coefficients k_{ab} are perturbations of coefficients $K^{(0)}{}_{ab}$ which give the geometry of the hypersurface of the maximum of the density of the energy for the unperturbed functions. The coefficients $K^{(0)}{}_{ab}$ for sector of the field theory localized around the straight-linear vortices are $K^{(\mathrm{cyl})}{}_{ab}$ (they give the geometry of the cylinder) or for the sector localized around the closed loops are $K^{(\mathrm{tor})}{}_{ab}$ and give the geometry of the torus. The coefficients $K_{ab} = K^{(0)}{}_{ab} + k_{ab}$ give the geometry of the perturbed surface.

Equations (5)–(8) give the geometric constraints which describe how any vortex solution can be deformed and remain a solution of field equations.

For example, for Abrikosov–Nielsen–Olesen solutions the membrane is a cylinder with an axis bent like a Nambu–Goto string ($K_i{}^i = 0$, where $(\sigma^i) = (\tau, \sigma)$ are string coordinates) but the radius of the hypersurface obtained by the deformation of straight cylinder has its own dynamics.

3. Excitations of closed loops

Let us consider a closed loop which is a Nielsen–Olesen-type configuration

$$\phi = \mathrm{e}^{-in\varphi} F(r_0 + \xi) \qquad A_\varphi = (r_0 + \xi)^2 A(r_0 + \xi) \qquad A_t = A_\theta = 0 \tag{10}$$

where $F(r_0 + \xi)$ and $A(r_0 + \xi)$ are some functions of ξ and $r_0 = r_0(t)$. The thickness of the loop is equal to $2r_0$ and $R = R(t)$ is the radius of the loop (see figure 1).

It has been proved numerically that closed loop collapses and dependence of R and r_0 on time is highly nonlinear (only for large loops acceleration is constant) [11]. Equations of the first order on ansatz (10) have the form:

$$\phi^1 k_a{}^a = 0 \tag{11}$$

$$g^{ab}(\nabla_a k_b{}^\varphi) = 0 \tag{12}$$

$$[k_{hc} K_{\mathrm{tor}}{}^{c\varphi} + \tfrac{1}{2} k_h{}^\varphi K_{\mathrm{tor}_c}{}^c + k^{a\varphi} K^{\mathrm{tor}}{}_{ah}] A^0{}_\varphi = k_h{}^\varphi A^1{}_\varphi \tag{13}$$

$$k^{\varphi\varphi} = 0. \tag{14}$$

Coefficients of the extrinsic curvature for torus are given by the formulae:

$$\begin{aligned} &K^{\mathrm{tor}}{}_{t\varphi} = K^{\mathrm{tor}}{}_{t\theta} = K^{\mathrm{tor}}{}_{\theta\varphi} = 0 \\ &K^{\mathrm{tor}}{}_{\varphi\varphi} = -f r_0 \qquad K^{\mathrm{tor}}{}_{\theta\theta} = f \dot{f} \qquad K^{\mathrm{tor}}{}_{tt} = f \cos(\varphi)(R + r_0 \cos(\varphi)) \end{aligned} \tag{15}$$

where $f = \dot{r_0} + \dot{R}\cos(\varphi)$, '$\cdot$' is derivative with respect to t.

Equations (13) for $h = t$, $h = \varphi$ and equation (14) form a system of algebraic equations on k_{tt},$k_{t\varphi}$,$k_{\varphi\varphi}$ with a trivial solution

$$k_{tt} = k_{t\varphi} = k_{\varphi\varphi} = 0. \tag{16}$$

Equation (11) is an algebraic equation on k_{tt}, $k_{t\varphi}$, $k_{\varphi\varphi}$ and $k_{\theta\theta}$ when put together with (16) yields:

$$k_{\theta\theta} = 0. \tag{17}$$

Now we have to deal with equations (12) and (13) for $h = \theta$. If we put (16)–(17) into equations (12) and (13) for $h = \theta$ we will obtain:

$$[g^{\theta\theta} g^{\varphi\varphi}]\partial_\theta k_{\theta\varphi} + [g^{\theta\theta} g^{t\varphi}]\partial_\theta k_{\theta t} = 0 \tag{18}$$

$$k_{\theta\varphi} = B k_{\theta t} \tag{19}$$

where B is a rather complicated function of ${K^{\mathrm{tor}}}_{ab}$, g^{ab} and their derivatives. Because g^{ab} does not depend on θ, B does not depend on θ either. In the limit $\dot{R} \ll 1$, $\dot{r_0} \ll 1$, $R \gg r_0$ the coefficient B takes form $B = r_0 \dot{R} \sin(\varphi)$. In the same limit coefficients of the extrinsic curvature of the torus up to terms linear in perturbations become

$$ {K^{\mathrm{tor}}}_{t\varphi} = {K^{\mathrm{tor}}}_{t\theta} = {K^{\mathrm{tor}}}_{\theta\varphi} = 0$$

$$ {K^{\mathrm{tor}}}_{\varphi\varphi} = -r_0 \qquad {K^{\mathrm{tor}}}_{\theta\theta} = \dot{f} \qquad {K^{\mathrm{tor}}}_{tt} = \cos(\varphi)(R + r_0 \cos(\varphi)).$$

If we put (19) into (18) we will obtain $\partial_\theta k_{\theta t} = 0$ e.g. $k_{\theta t}$ is arbitrary but does not depend on θ. If $k_{\theta t}$ is first order then in approximation to the first order, because of the form of B, the coefficient $k_{\theta\varphi}$ is equal to zero $k_{\theta\varphi} \approx 0$. Thus, we have obtained that the perturbed solution is described by the surface characterized by extrinsic curvature coefficients of the torus $K_{ab} = {K^{\mathrm{tor}}}_{ab}$ except $K_{\theta t} = {K^{\mathrm{tor}}}_{\theta t} + k_{\theta t}$ where $k_{\theta t}$ is an arbitrary function which does not depend on θ. It is easy to show that conditions on K_{ab} are fulfilled by the surface of the bubble excited torus

$$[X^\mu] = [t, (R + r_0\psi \cos(\varphi)) \cos(\theta), (R + r_0\psi \cos(\varphi)) \sin(\theta), r_0\psi \sin(\varphi)] \tag{20}$$

if $\psi = \psi(\theta + \omega t)$, $\omega = \frac{1}{R}$ is only, small first order in $\frac{r_0}{R}$, perturbation of the toroidal surface (e.g. infinitesimally differs from one). The only free parameter for this surface is equal to $k_{\theta t} = (\dot{r_0}\acute{\psi} + \frac{r_0}{R}\psi'') \cos^2(\varphi)$, ‘′’ is derivative with respect to θ.

The bubble-like solutions we obtained above (see figure 2) are different from solutions obtained in previous work [10]. We obtained bubbles which do not move with constant velocity along a straight line but moving around the loop which radius R decreases in nonlinear fashion with time. The trajectory of the bubble moving with the speed of light has the shape of a spiral. The second effect which is absent in the case of the bubble moving along the straight line is contraction of the size of the bubble which is a consequence of decreasing of the thickness of the torus $r_0(t)$.

More careful investigations seem to show that if excitations, found in this section, are small enough then radiation is negligible [12].

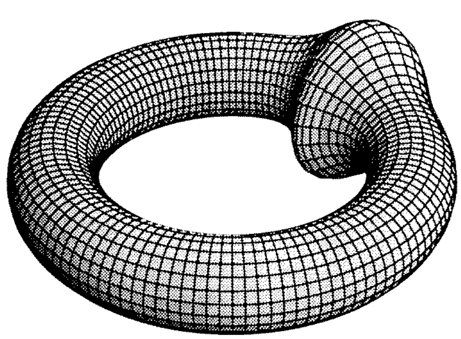

Figure 2. Bubble-like excitation on the vortex loop.

4. Tracks of the $\frac{\pi}{2}$ scattering in the membrane equations

The next interesting effect which is transparent in the membrane method is the $\frac{\pi}{2}$ scattering. It has been examined that the $\frac{\pi}{2}$ scattering appears not only for local strings (in the Abelian Higgs model) but also for global strings (in the Goldstone model) so it is a consequence of selfinteractions of the scalar field. More precise analysis shows that the $\frac{\pi}{2}$ scattering is a consequence of the short-range interactions. This is why we can restrict ourself to considerations of the first-order equation in perturbation ε for the scalar field (5)

$$\phi^1 k_a{}^a = 0. \tag{21}$$

In the case of the vortex solution with $n = 2$, $\phi = \mathrm{e}^{-2\mathrm{i}\theta} F(r_0 + \xi, \bar{\varepsilon}(t))$ where $\bar{\varepsilon}(t)$ is a distance of the line of zeros of the Higgs field from the centre of the mass of the system. For small distances $F \sim F_{-\bar{\varepsilon}} F_{\bar{\varepsilon}} \sim \sqrt{(x-\bar{\varepsilon})^2 + y^2}\sqrt{(x+\bar{\varepsilon})^2 + y^2}$ using definition $r = \sqrt{x^2 + y^2} = r_0 + \xi$ we can evaluate $\phi^1 \neq 0$. The elements of the metric tensor on the cylinder are $g^{ij} = \eta^{ij}$, $g^{i\theta} = 0$, $g^{\theta\theta} = -r_0{}^2$ so equation (22) takes the form:

$$k_{tt} - k_{zz} - r_0{}^2 k_{\theta\theta} = 0. \tag{22}$$

Let us consider the slightly ($\varepsilon \ll r_0$) elliptically deformed cylindrical surface

$$[X^\mu] = [t, (r_0 + \varepsilon)\cos(\theta), (r_0 - \varepsilon)\sin(\theta), z]. \tag{23}$$

For such surface, in the limit $r_0 \gg \varepsilon$, we can easily evaluate coefficients of the extrinsic curvature

$$\begin{aligned} &K_{tz} = K_{zz} = K_{\theta\theta} = 0 \\ &K_{t\theta} = -\dot{\varepsilon}\sin(2\theta) \qquad K_{\theta\theta} = -r_0 - \varepsilon = K^{\mathrm{cyl}}{}_{\theta\theta} - \varepsilon \qquad K_{tt} = \ddot{\varepsilon}. \end{aligned} \tag{24}$$

Using the definition of the k_{ab} ($K_{ab} = K^{\mathrm{cyl}}{}_{ab} + k_{ab}$) we can evaluate perturbations of the coefficients of the extrinsic curvature, of the elliptically deformed cylinder, from cylindrical surface. They are equal to

$$k_{t\theta} = -\dot{\varepsilon}\sin(2\theta) \qquad k_{\theta\theta} = -\varepsilon \qquad k_{tt} = \ddot{\varepsilon} \tag{25}$$

remaining k_{ab} are equal to zero. Now if we put (26) into (23) we will obtain

$$\ddot{\varepsilon} + r_0{}^2\varepsilon = 0. \tag{26}$$

If we choice the simple solution of the equation (27) $\varepsilon(t) = \varepsilon_0\cos(r_0 t)$ then we can write *in* and *out* states of the membrane

$$[X^\mu]_{\mathrm{in}} = [0, (r_0 + \varepsilon_0)\cos(\theta), (r_0 - \varepsilon_0)\sin(\theta), z] \tag{27}$$

$$[X^\mu]_{\mathrm{out}} = \left[\frac{\pi}{r_0}, (r_0 - \varepsilon_0)\cos(\theta), (r_0 + \varepsilon_0)\sin(\theta), z\right]. \tag{28}$$

The section of the surface $[X^\mu]_{\mathrm{in}}$ in the X–Y plane gives ellipse prolate in x-direction and section $[X^\nu]_{\mathrm{out}}$ gives ellipse prolate in y-direction. We can transform each other by coordinate transformation $x \to y, y \to x$. Although we do not know explicit dependence position of the zeros $\bar{\varepsilon}$ on deformation of the cylindrical surface ε e.g. $\bar{\varepsilon} = f(\varepsilon)$, having $F_{\mathrm{in}} \sim \sqrt{(x - \bar{\varepsilon_0})^2 + y^2}\sqrt{(x + \bar{\varepsilon_0})^2 + y^2}$ we can by transformation $x \to y, y \to x$, obtain out field state $F_{\mathrm{out}} \sim \sqrt{(y - \bar{\varepsilon_0})^2 + x^2}\sqrt{(y + \bar{\varepsilon_0})^2 + x^2}$.

Equation (27) says that ellipse prolate along the x-axis after half of the period turns into ellipse prolate along the y-axis. It means that at first zeros of the Higgs field come to each other along the x-axis and then start running away along the y-axis (see figure 3). According to expectations we obtained that the $\frac{\pi}{2}$ scattering is due to the short-range interactions of the scalar field. We can worry only about the fact that after half of the period zeros of the

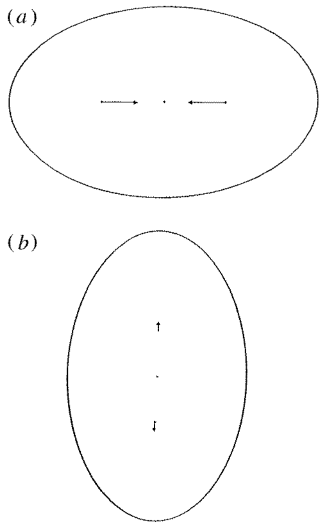

Figure 3. (*a*) Zeros of the Higgs field move to oneself along the x-axis. (*b*) Zeros of the Higgs field run away along the y-axis.

Higgs field start running in the opposite direction—this is a consequence of the fact that the membrane method describes only bound states. We can easily imagine that if the initial speed of the zeros is sufficiently large the approximation fails and the course of the process of the scattering finishes with the two zeros running away into plus and minus infinity along the y-direction. If we let dependence ε on t and z then equation (27) takes the form $\ddot{\varepsilon} - \varepsilon'' + {r_0}^2\varepsilon = 0$, which describes travelling waves moving along the vortex [13].

5. Remarks

With a help of the membrane method of constructing excited vortex solutions in the Abelian Higgs model we have investigated two physical situations.

First, we considered bubble-like excitations of the closed vortex loop. We obtained that for large enough loops collapsing with nonrelativistic speed there is a possibility of existing small bubble-like excitations moving along the loop with the speed of light. Excitations found in this paper differ from those which move along the straight vortex [10]—they move along the collapsing circle.

The second phenomenon is the $\frac{\pi}{2}$ scattering. We have confirmed that the $\frac{\pi}{2}$ scattering is a consequence of short-range interactions of the scalar field. On the other hand, this result may be treated as a test of accuracy of the membrane method.

Acknowledgments

I would like to thank H Arodź for discussions and J Dziarmaga for calling my attention to the fact that the $\frac{\pi}{2}$ scattering may be transparent in the membrane equations. Work partly supported by the grant KBN 2 P302 049 05.

References

[1] Zeldovich Y B 1980 *Mon. Not. R. Astron. Soc.* **192** 663
Vilenkin A and Field G 1987 *Nature* **326** 722
Vilenkin A 1981 *Phys. Rev.* D **23** 852
[2] Brout R, Englert F and Fishler W 1976 *Phys. Rev. Lett.* **36** 649
't Hooft G 1979 *Nucl. Phys.* B **153** 141
Baker M, Ball J S and Zachariasen F 1988 *Phys. Rev.* D **38** 1926
[3] Lund F and Regge T 1976 *Phys. Rev.* D **14** 1524
Abrikosov A A 1957 *Zh. Exp. Theor. Phys.* **32** 1442
[4] Arodź H and Węgrzyn P 1992 *Phys. Lett.* B **291** 251
[5] Nielsen H B and Olesen P 1973 *Nucl. Phys.* B **61** 45
[6] Föster D 1974 *Nucl. Phys.* B **81** 84
[7] Ruback P J and Shellard E P S 1988 *Phys. Lett.* **209B** 262
Myers E, Rebbi C and Strilka R 1988 *Phys. Lett.* **209B** 262
Dziarmaga J 1993 *Phys. Rev.* D **48** 3809
[8] Silveira V 1990 *Phys. Rev.* D **41** 1914
[9] Ben-Ya'acov U 1992 *Nucl. Phys.* B **382** 597
[10] Dobrowolski T 1994 *Phys. Rev.* D **50** 6503
[11] de Vega H 1977 *preprint* PAR LPTHE 7714
[12] Arodź H and Hadasz L 1996 *Phys. Rev.* D **54** 4004
[13] Dziarmaga J 1994 *Phys. Lett.* **328B** 392